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M.Sc. (Sem. - III) Examination October - 2025

Mathematics : Advanced Numerical Analysis (Paper - PGMTH - 302)

Time: 3 Hours] **[Total Marks: 70**

સૂચના : / Instructions

(1) ઉપરોક્ત દર્શાવેલ વિગતો ઉત્તરવહી પર અવશ્ય લખવી.
Fill up strictly the above details on your answer book

(2) All questions are compulsory

(3) Figures to the right indicate marks of the corresponding question

Seat No.:

Student's Signature

Q. 1. **Answer the following (any two)** **14**

1. Derive the Chebyshev Method for solving the equation $f(x) = 0$.
2. Perform three iterations of the Muller method to find the root of the equation $f(x) = x^3 - \frac{1}{2}$. Use the initial approximation $x_0 = 0$, $x_1 = 0.5$ and $x_2 = 1$.
3. Find the real root of the equation $3x^3 + 4x^2 + 4x + 1 = 0$ by using the general iteration method, correct up to three decimal places.

Q. 2. **Answer the following (any two)** **14**

1. Solve the following system of equations using the Gauss elimination method with partial pivoting

$$9x_1 - x_2 + 2x_3 = 4,$$
$$x_1 + 10x_2 - x_3 = 3,$$
$$2x_1 + 3x_2 + 20x_3 = 7$$

2. Find the inverse of the $A = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 3 & 2 \\ 1 & 2 & 2 \end{bmatrix}$ by using Crout's method.
3. Using the Gauss-Seidel iteration scheme in matrix form to solve the system of equations. Start with initial approximation $x^{(0)} = [0.5 - 0.5 - 0.5]^T$ and perform three iterations

$$\begin{bmatrix} 2 & 3 & 1 \\ 3 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ x \end{bmatrix} = \begin{bmatrix} -1 \\ 1 \\ 6 \end{bmatrix}$$

Q. 3. **Answer the following (any two)** **14**

1. Discuss the numerical differentiation method based on interpolation for uniform nodal points for linear and quadratic interpolation.
2. Find the smallest eigenvalue nearest to 3 in magnitude and the corresponding eigenvector of the matrix $A = \begin{bmatrix} 4 & 1 & 0 \\ 1 & 4 & 1 \\ 0 & 1 & 4 \end{bmatrix}$ by using the four iterations of the inverse power method with an initial approximation $[1\ 1\ 1]^T$

3. Given the following values of $f(x) = x^4$, find the approximate value of $f'(0.8)$ and $f''(0.8)$ using quadratic interpolation. Also, obtain an upper bound on the error.

x	0.4	0.6	0.8
$f(x)$	0.0256	0.1296	0.4096

Q. 4. Answer the following (any two) **14**

1. Find the Quadrature formula $\int_0^1 \frac{f(x)}{\sqrt{x(1-x)}}\,dx = \alpha_1 f(0) + \alpha_2 f\left(\frac{1}{2}\right) + \alpha_3 f(1)$, which is exact for polynomials of the highest possible degree. Then use the formula on $\int_0^1 \frac{f(x)}{\sqrt{x-x^3}}\,dx$.
2. Find the remainder of the Simpson's $\left(\frac{3}{8}\right)^{th}$ rule $\int_{x_0}^{x_3} f(x)\,dx = \frac{3h}{8}[f(x_0) + 3f(x_1) + 3f(x_2) + f(x_3)]$ for equally spaced points $x_i = x_0 + ih$, $i = 1, 2, 3$. Using Simpson's $\left(\frac{3}{8}\right)^{th}$ rule, approximate the value of the integral $I = \int_0^1 \frac{dx}{1+x}$.
3. Evaluate the integral $I = \int_{-1}^{1} (1 - x^2)^{\frac{3}{2}} \cos x\,dx$ using the Gauss-Chebyshev one-point, two-point and three-point quadrature rules. Also, evaluate using the Gauss-Legendre three-point formula.

Q. 5 Answer the following (any two) **14**

1. Using modified Euler-Cauchy method to solve the initial value problem on the interval [0,0.4] for $u' = -2tu^2$, $u(0) = 1$ with $h = 0.2$. Also estimate $u(0.4)$.
2. Using the second order finite difference method to solve the boundary value problem $u'' = xu$, $u(0) + u'(0) = 1$, $u(1) = 1$ with $h = \frac{1}{3}$.
3. Solve the system of equation $u' = -3u + 2v, u(0) = 0$ and $v' = 3u - 4v, v(0) = 0.5$ with $h = 0.2$ on the interval [0,0.4] by using Heun method.